I SEMESTER EXAMINATION 2017-2018

SUBJECT: MATHEMATICS & STATISTICS

STD:XII SCI — MAX MARKS:80

DATE:09/10/17 — TIME: 3HRS

Note:

i. All questions are compulsory.

ii. Figures to the right indicate full marks.

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)**

i. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A[illegible] = _______.

(A) 6A (B) 12A (C) 16A (D) 32A

ii The principal solution of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{2}$

(ii) If $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$, then the value of k is ___________.

(a) $\frac{1}{2}$ (b) $\frac{11}{2}$ (c) $\frac{5}{2}$ (d) $\frac{-11}{2}$

(B) Attempt any THREE of the following: **(6)**

i. Write truth values of the following statements:

a. $\sqrt{5}$ is an irrational number but $3 + \sqrt{5}$ is a complex number

b. $\exists n \in N$ such that $n + 5 > 10$

ii. In Δ ABC, prove that, $a(b \cos C - c \cos B) = b^2 - c^2$.

ii. Find 'k', if the sum of slopes of lines represented by equation $x^2 + kxy - 3y^2 = 0$ is twice their product.

(i) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and AX = I, then find X by elementary transformation.

v. Write the dual of the following statements:

a. $(p \vee q) \wedge T$

b. Madhuri has curly hair and brown eyes.

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Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$, then find the value of 'x'.

ii. Write the converse, inverse and contrapositive of the following statement.
"If it rains then the match will be cancelled."

iii. Find p and q, if the equation
$px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of prependicular lines.

(B) Attempt any TWO of the following: **(8)**

i. In ΔABC with the usual notations prove that
$(a - b)^2 \cos^2\left(\frac{C}{2}\right) + (a + b)^2 \sin^2\left(\frac{C}{2}\right) = c^2$.

ii. Find the joint equation of the pair of lines passing through the origin which are perpendicular respectively to the lines represented by $5x^2 + 2xy - 3y^2 = 0$.

iii. The sum of three numbers is 6. When second number is subtracted from thrice the sum of first and third number, we get number 10. Four times the third number is subtracted from five times the sum of first and second number, the result is 3. Using above information, find these three numbers by matrix method.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. In ΔABC with usual notations, prove that
$2\left\{a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}\right\} = (a + c - b)$

ii. If p : It is a day time, q : It is warm, write the compound statements in verbal form denoted by–

a. $p \wedge \sim q$

b. $\sim p \rightarrow q$

c. $q \leftrightarrow p$

i. Using truth tables, examine whether the statement pattern $(p \wedge q) \vee (p \wedge r)$ is a tautology, contradiction or contingency.

(B) Attempt any TWO of the following: **(8)**

i. In any ΔABC, with usual notations, prove that
$b^2 = c^2 + a^2 - 2ca \cos B$

ii. Show that the equation $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ represents a pair of lines. Find the acute angle between them. Also find the point of intersection of the lines.

iii. Express the following equations in the matrix form and solve them by the method of reduction:
$2x - y + z = 1, x + 2y + 3z = 8, 3x + y - 4z = 1$.

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SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If the function

$f(x) = k + x$, for $x < 1$

$= 4x + 3$, for $x \geq 1$

is continuous at $x = 1$ then k =

(A) 7 (B) 8 (C) 6 (D) –6

(i) Function $f(x) = x^2 - 3x + 4$ has minimum value at $x =$ ________.

(a) 0 (b) $-\frac{3}{2}$ (c) 1 (d) $\frac{3}{2}$

(ii) $\int \frac{1}{x} \cdot \log x\, dx =$ ________________.

(a) $\log(\log x) + c$ (b) $\frac{1}{2}(\log x)^2 + c$ (c) $2\log x + c$ (d) $\log x + c$

(B) Attempt any THREE of the following: **(6)**

i. If $y = x^x$, find $\frac{dy}{dx}$.

ii. The displacement 's' of a moving particle at time 't' is given by $s = 5 + 20t - 2t^2$. Find its acceleration when the velocity is zero.

iii. Evaluate: $\int \sec^n x \cdot \tan x\, dx$

i. If $y = \sin^{-1}(3x) + \sec^{-1}\left(\frac{1}{3x}\right)$, find $\frac{dy}{dx}$.

ii. Evaluate: $\int x \log x\, dx$.

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Examine the continuity of the following function at given point:

$f(x) = \frac{\log x - \log 8}{x - 8}$, for $x \neq 8$

$= 8$, for $x = 8$ at $x = 8$

(ii) If $x = a\left(t - \frac{1}{t}\right), y = a\left(t + \frac{1}{t}\right)$, then show that $\frac{dy}{dx} = \frac{x}{y}$.

ii. Evaluate : $\int \frac{(x+1)}{(x-2)(x+3)} dx$

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(B) Attempt any TWO of the following: **(8)**

i. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ where $\frac{dy}{dx} \neq 0$.

Hence find $\frac{d}{dx}\left(\tan^{-1} x\right)$.

ii. A telephone company in a town has 5000 subscribers on its list and collects fixed rent charges of ₹ 3,000 per year from each subscriber. The company proposes to increase annual rent and it is believed that for every increase of one rupee in the rent, one subscriber will be discontinued. Find what increased annual rent will bring the maximum annual income to the company.

iii. If the function f(x) is continuous in the interval [–2, 2], find the values of a and b, where

$$f(x) = \frac{\sin ax}{x} - 2, \quad \text{for } -2 \le x < 0$$
$$= 2x + 1, \quad \text{for } 0 \le x \le 1$$
$$= 2b\sqrt{x^2 + 3} - 1, \quad \text{for } 1 < x \le 2$$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Examine the maxima and minima of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. Also, find the maximum and minimum values of f(x).

ii. Evaluate : $\int \frac{1}{3 + 2\sin x + \cos x} dx$

iii. If $x^p y^q = (x + y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

(B) Attempt any TWO of the following: **(8)**

i. Evaluate: $\int \frac{1}{x \log x \log(\log x)} dx$

ii. Prove that:

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

ii. Verify Lagrange's mean value theorem for the function

$f(x) = x + \frac{1}{x}, x \in [1, 3]$

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