I SEMESTER EXAMINATION 2017-2018

SUBJECT: MATHEMATICS & STATISTICS

STD:XII COM MAX MARKS:80

DATE:07/10/17 TIME : 3HRS

Notes:

i. All questions are compulsory.

ii. Figures to the right indicate full marks.

iii. Answer to every question must be written on a new page.

Section – I

Q.1. Attempt any SIX of the following: [12]

i. If $y = (\sin x)^x$, find $\frac{dy}{dx}$.

ii. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ show that $A^2 - 2A$ is a scalar matrix.

iii. If $A = \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, then verify that $|AB| = |A||B|$

iv. Discuss the continuity of the function at the point given. If the function is discontinuous then remove the discontinuity.

$$f(x) = \frac{\sin^2 5x}{x^2}, \quad \text{for } x \neq 0$$

$$= 5, \quad \text{for } x = 0, \text{ at } x = 0$$

vi. Find $\frac{dy}{dx}$, if $y = x^{e^x}$

vi. Differentiate: $\tan^{-1}(\cot 2x)$ w.r.t.x.

vii. Discuss the continuity of function f at $x = 0$

Where $f(x) = \frac{\sqrt{4+x}-2}{3x}$, for $x \neq 0$

$= \frac{1}{12}$, for $x = 0$

viii. State which of the following sentences are statements. In case of statement, write down the truth value:

(a) Every quadratic equation has only real roots.

(b) $\sqrt{-4}$ is a rational number.

Q.2. (A) Attempt any TWO of the following: **[6][14]**

i. Solve the following equations by the inversion method:

$2x + 3y = -5$ and $3x + y = 3$ (3)

ii. Find x and y, if $\left\{3\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & 3\end{bmatrix}-\begin{bmatrix}1 & 5 & -2\\ -3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$ (3)

ii. If $y = \tan^{-1}\left[\dfrac{\cos 2x - \sin 2x}{\sin 2x + \cos 2x}\right]$ then find $\dfrac{dy}{dx}$. (3)

(B) Attempt any TWO of the following: **[8]**

i. Using the truth table verify that

$p \vee (q \wedge r) - (p \vee q) \wedge (p \vee r)$. (4)

i. If the function

$f(x) = x^2 + ax + b, \quad x < 2$

$= 3x + 2, \quad 2 \le x \le 4$

$= 2ax + 5b, \quad 4 < x$

is continuous at $x = 2$ and $x = 4$, then find the values of a and b. (4)

i. Find the inverse of the following matrix by elementary row transformations if it exists.

$$A = \begin{vmatrix}1 & 2 & -2\\ 0 & -2 & 1\\ 1 & 3 & 0\end{vmatrix}$$ (4)

Q.3. (A) Attempt any TWO of the following: **[6][14]**

i. Examine whether the following statement $(p \wedge q) \vee (\sim p \vee \sim q)$ is a tautology or contradiction or neither of them. (3)

ii. If the function

$f(x) = \dfrac{15^x - 3^x - 5^x + 1}{x \tan x}$, $x \neq 0$ is continuous at $x = 0$, then find $f(0)$. (3)

ii. Examine the continuity of the following function:

$f(x) = x^2 - x + 9, \quad \text{for } x \le 3$

$= 4x + 3, \quad \text{for } x > 3$ } at $x = 3$ (3)

(B) Attempt any TWO of the following: **[8]**

i. (a) Express the truth of each of the following statements using Venn diagram.

(1) All teachers are scholars and scholars are teachers.

(2) If a quadrilateral is a rhombus then it is a parallelogram.

(b) Write converse and inverse of the following statement:

"If Ravi is good in logic then Ravi is good in Mathematics." (4)

ii. If $x^3y^5 = (x + y)^8$, then show that $\dfrac{dy}{dx} = \dfrac{y}{x}$ (4)

iii. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = 2k$,

show that $y\frac{dy}{dx} = x\tan^2 k$. (4)

Section – II

Q.4. Attempt any SIX of the following: **[12]**

i. The ratio of number of boys and girls in a school is 3 : 2. If 20% of the boys and 30% of the girls are scholarship holders, find the percentage of students who are not scholarship holders. (2)

ii. Calculate crude death rates (CDR) for district A:

District A

Age groups (in years)	Number of persons (in thousands)	Number of death
0 – 15	1	20
15 – 60	3	30
60 and above	2	40

(2)

iii. Compute the coefficient of correlation for the following data:

$n = 100,\ \bar{x} = 62,\ \bar{y} = 53,\ \sigma_X = 10,\ \sigma_Y = 12,$

$\sum(x_i - \bar{x})(y_i - \bar{y}) = 8000$ (2)

iv. A building is insured for 80% of its value. The annual premium at 70 paise percent amounts to ₹ 2,800. Fire damaged the building to the extent of 60% of its value. How much amount for damage can be claimed under the policy? (2)

v. For an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is ₹ 10,000. What is its accumulated value after 3 years.
[Given: $(1.1)^3 = 1.331$]. (2)

vi. A, B and C are in partnership. A's capital was ₹ 65,000. C's capital was ₹ 50,000. The total profit is ₹ 38,000, out of which B's profit is ₹ 15,000. What was B's capital? (2)

vii. Draw scatter diagram for the following data and interpret it:

x	10	20	30	40	50	60	70
y	32	20	24	36	40	28	38

(2)

viii. If $\Sigma d^2 = 66$ and $n = 10$ then find the rank correlation coefficient. (2)

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Find the present value of annuity immediate of ₹ 18,000 p.a. for 3 years at 9% p.a. compounded annually. [Given: $(1.09)^{-3} = 0.7722$] (3)

ii. Complete the following life table:

x	l_x	d_x	q_x	p_x	L_x
4	9100	60	?	?	?
5	?	45	?	?	

ii. Calculate CDR for districts A and B and compare them.
Also state which district is more healthy. (3)

Age group (in years)	District A		District B	
	No. of Persons ('000)	No. of Deaths	No. of Persons ('000)	No. of Deaths
0 – 15	1	20	2	50
15 – 60	3	30	7	70
60 and above	2	40	1	25

(B) Attempt any TWO of the following: **(8)**

i. A bill of ₹ 2,000 drawn on 15th February 2003 for 10 months was discounted on 13th May 2003 at $3\frac{3}{4}$ % p.a. Calculate the banker's discount. (4)

ii. In the following data, one of the values of Y is missing. Arithmetic means of X and Y series are 6 and 8 respectively.

X	6	2	10	4	8
Y	9	11	?	8	7

a. Estimate the missing observation.
b. Calculate correlation coefficient. (4)

i. John and Mathew started a business with their capitals in the ratio 8 : 5. After 8 months, John added 25% of his earlier capital as further investment. At the same time, Mathew withdrew 20% of his earlier capital. At the end of the year, they earned ₹ 52,000 as profit. How should they divide the profit between them? (4)

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Compute the age specific death rate for the following data:

Age groups (years)	Population (in thousands)	Number of deaths
Below 5	15	360
5 – 30	20	400
Above 30	10	280

(3)

ii. A bill of ₹ 7,500 was discounted for ₹ 7,290 at a bank on 28th October 2006. If the rate of interest was 14% p.a., what is the legal due date of the bill? (3)

ii. Mr. Natarajan and Mr.Gopalan are partners in the company having capitals in the ratio 4 : 5 and the profits received by them are in the ratio 5 : 4. If Mr. Gopalan invested capital in the company for 16 months, how long was Mr. Natarajan's investment in the company? (3)

(B) Attempt any TWO of the following: **(8)**

i. Ranking of 8 trainees at the beginning (X) and at the end (Y) of a certain course are given below.

Trainees	A	B	C	D	E	F	G	H
X	1	2	4	5	6	8	3	7
Y	2	4	3	7	8	1	5	6

ii. The following data gives the marks of 20 students in Mathematics (X) and Statistics (Y), each out of 10, expressed as (X, Y). Construct ungrouped frequency distribution considering single number as a class. Also prepare marginal distributions
(2, 7), (3, 8), (4, 9), (2, 8), (2, 8), (5, 6), (5, 7), (4, 9), (3, 8), (4, 8), (2, 9), (3, 8), (4, 8), (5, 6), (4, 7), (4, 7), (4, 6), (5, 6), (5, 7), (4, 6). (4)

i. From the following data, find crude death rates (C.D.R.) for Town I and Town II, and comments on the results:

Age group	Town I		Town II	
(years)	Population	No. of deaths	Population	No. of deaths
0 – 10	1500	45	6000	150
10 – 25	5000	30	6000	40
25 – 45	3000	15	5000	20
45 and above	500	22	3000	54

(4)